TOMOGRAPHIC INVERSION OF TRAVEL TIME IN TERMS OF DEPTH PARAMETERS

J.A. Madrid and J.C.V. Traslosheros
Ciencias de la Tierra
Departamento de Sismología y
Departamento de Geofísica Aplicada
Fax: 011 52 (667) 4 49 33
CICESE

ABSTRACT

Un this study a method was developed for generating the partial derivatives of the travel time with respect to depth for an isovelocity line from a bidimensional model with lateral variation. This allows to solve the linearized Inverse System, using both parameters (velocity and depth) sequentially or simultaneously. For the calculation of travel time, slowness and derivatives, the media was divided in triangular regions, where the propagation velocity gradient of the waves is constant.

For the inversion procedure, SVD was used with regularization. The method can be used to locate discontinuites of irregular topography. Samples are presented with real and synthetic data.

INTRODUCTION

Waves traveling through a medium carry information that can be inferred from the behavior of the waveshape and of its arrival time at the receiver. Even if the travel times do not contain as much information as the singal, a good deal of inference can be made by looking at them. The problem of determining the structure of velocity of a medium using only the travel time of internal seismic waves has been thoroughly treated in the last twenty years (see for example Wiggins (1972), Jackson (1972), Aki (1977)). Some methods have been developed to invert in terms of velocities and source parameters (Spencer and Gubbins, 1980). Some other methods use as parameters the velocities and the depth of discontinuities (Jones and Jovanovitch (1985), Bishop et al., (1985), Huang et al (1986)).

In this paper we present a method to invert travel time data using a mixed set of point parameters that define both the velocity and the vertical position of isovelocities. The inversion is performed using singular value decomposition (SVD) applied over a minimum squared regularized scheme (Traslosheros et al., 1990) To compute the partial derivative, we use a ray tracing technique called the "circular approximation". In this technique, the medium is divided in triangular regions where in the gradient of velocity is constant, so that in every triangle the local horizontal component of the slowness vector is conserved. This feature makes the computation ot the matrix of derivatives straightforward. Departing from known expressions for the derivative with respect to the velocity, we implement a transformation that exchanges the roles of velocity and depth, considering the velocity as a coordinate. This kind of parameterization allows the introduction of constrictions that are helpful in the search of discontinuities.

THEORY

It is well known that in an isotropic medium with velocity.

$$u = u_0 + b_1 x + b_2 z \qquad (1)$$

the rays are arcs of circles of radius $R = -1/p'b$, where $b = (b^2_1 + b^2_2)^{1/2}$ is the magnitude of the gradient of velocity. The local parameters of the ray are: the arc length S and the slowness vector (p,q). The gradient is a vector that defines a rotation to a new system of coordinates (x',z') where the slowness vector has new components(p',q') such that p is constant. In this system, the velocity becomes.

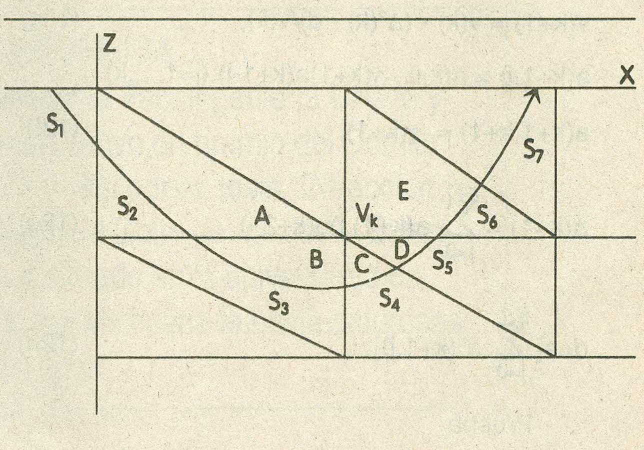

Figure 1. Triangular discretization grid.

$$u = bz' \quad (2)$$

where the free terms has been absorbed in the left side of. These properties are used to trace rays from a source to a receiver through a set of triangles that are obtained by discretizing the medium in a number of points (x_i, z_i) to each of which a value u_i of the velocity is assigned, as illustrated in figure 1. In this parameterization, the inversion of travel times is performed applying SVD to the equations

$$\delta t_i = \frac{\partial t_i}{\partial \upsilon_j} \partial \upsilon_j \quad (3)$$

The derivative in (3) takes the form (Madrid (1986))

$$\frac{\partial t_i}{\partial u_j} = \sum_{T_i} \frac{\partial \Delta t_{Ti}}{\partial \upsilon_j} \quad (4)$$

where the travel time inside a triangle (Δt_{Ti}) is

$$\Delta t_{Ti} = \int \frac{ds_{Ti}}{\upsilon(x,z)} \quad (5)$$

here, ΔT and Ti represent all the triangles that share the common point $\upsilon_i(x_i, z_i)$. Now, if we consider the rotated system (x',z'), it is easy to see that we can trace rays in either of two systems: the one that is gotten through the rotation, (x',z'), or a system with horizontal axis x and vertical axis v. The fact that v may be considered as a coodinate, is readily seen by writing (1) as

$$z = z_0 + a_1 x + a_2 u \quad (6)$$

where $z_0 = \upsilon_0 / b_2, a = -b_1/b_2, a_2 = 1/b_2$. This transformation is one-to-one so that to each couple (x,v) corresponds one and only one value of z. After a rotation by the angle r, the expression for the travel time inside a triangle becomes:

$$\frac{1}{b} \int \frac{ds}{z'} \quad (7)$$

Since b is constant, (7) has essentially the same form as (5). We see that the problem of inversion is equivalent when formulated either in terms of z', or in terms of v. For a triangle, we can write:

$$\delta(\Delta T) = \frac{\partial(\Delta t)}{\partial \upsilon} \delta\upsilon = \frac{\partial(\Delta t)}{\partial z'} \delta z' \quad (8)$$

but we wish to explain the time residuals $\delta(\Delta t)$ having as variable the z-coordinate while keeping x fixed, so that we can write:

$$\delta(\Delta T) = \frac{\partial(\Delta t)}{\partial z} \delta z \quad (9)$$

from (8) and (9), we obtain

$$\frac{\partial(\Delta t)}{\partial z} = -b_2 \frac{\partial(\Delta t)}{\partial \upsilon} \quad (10)$$

where we have used $\delta z' = \delta z \cos r$ and $r = -\tan^{-1}(b^1 / b^2)$ (figure 2).

Finally, in a fashion similar to equation (4), the z-derivative of the total travel time is

$$\frac{\partial t}{\partial z_i} = \sum_{Ti} \frac{\partial \Delta t_{Ti}}{\partial z_i} \quad (11)$$

A problem that could be critical emerges when we attemp the inversion using mixed (υ and z) parameters corresponding to the same point. However, it is easy to show that total linear dependence between related columns of the matrix is broken when we consider lateral heterogeneity. Finally, the z-parameterization allows two new things: 1) we can combine the use of mixed parameters corresponding to the same point, and 2) we can add conditions by operating directly on the variations of depth, as we will see in the following section.

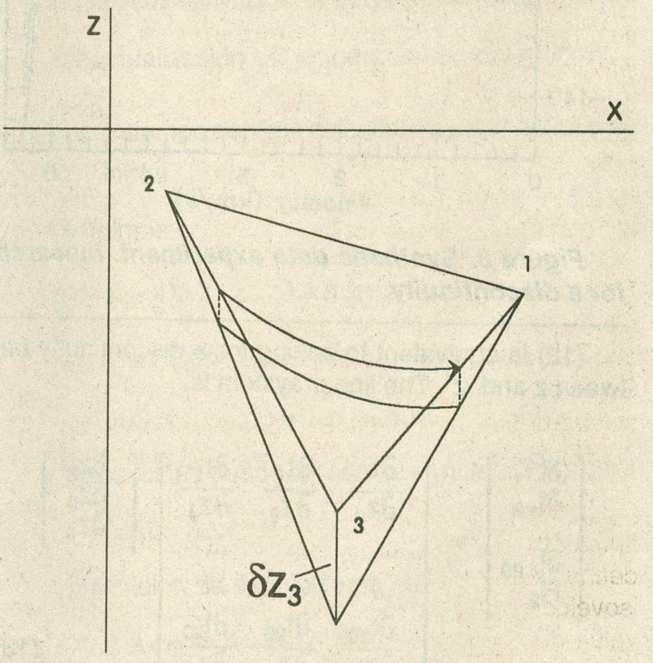

Figure 2. The perturbation δz causes a deformation of triangle.

EXAMPLES

The first example is the search of a discontinuity, departing from a smooth model and the raw data from rays bottoming in the lower medium. The "actual" model is shown in figure 3, together with the results of the first, fifth and seventh interation. In this example we use twenty travel times, and three z-points as parameters. To this we added the condition

$$H_2 = \delta z_3 - \delta z_2 \qquad (12)$$

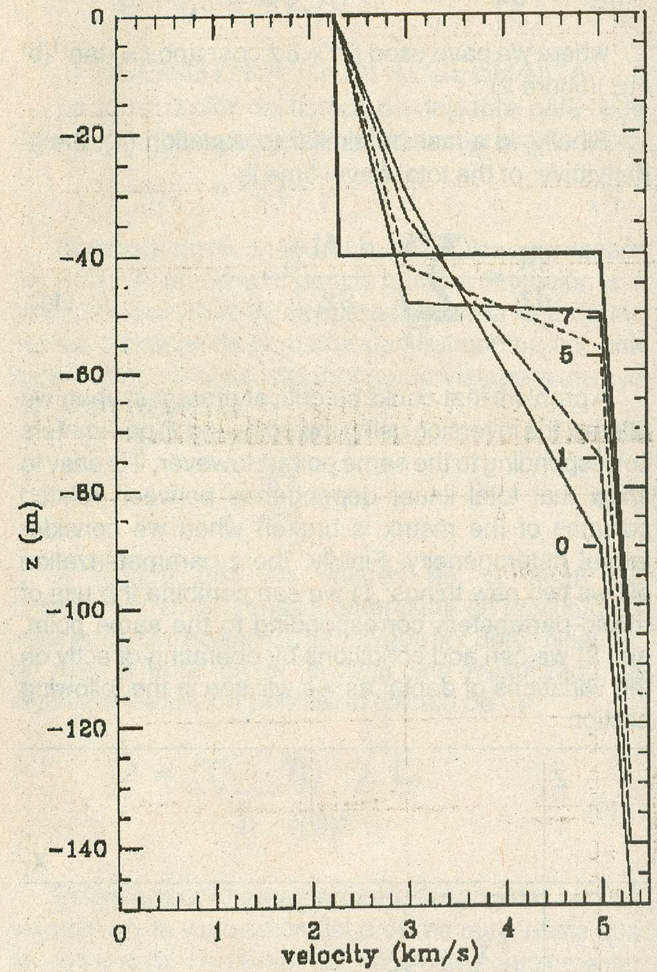

Figure 3. Synthetic data experiment. Insearch for a discontinuity.

(12) is equivalent to asking for a discontinuity between z and z . The linear system is

$$\begin{bmatrix} \delta t_1 \\ \delta t_2 \\ \dots \\ \delta t_{20} \\ H_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial t_1}{\partial z_2} & \frac{\partial t_1}{\partial z_3} & \frac{\partial t_1}{\partial z_4} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \frac{\partial t_{20}}{\partial z_2} & \frac{\partial t_{20}}{\partial z_3} & \frac{\partial t_{20}}{\partial z_4} \\ -1 & +1 & 0 \end{bmatrix} \begin{bmatrix} \delta z_2 \\ \delta z_3 \\ \delta z_4 \end{bmatrix} \qquad (13)$$

The results of the seventh interation may be improved adding more data to (13). Although the fifth iteration yields better residuals, the seventh represents a discontinuity. The second example is given in figure 4. Here, we simulated a basin shaped discontinuity with velocities 2.3 and 5.5 (km/s) just above and below the discontinuity. The travel time for an explosion at the origin yielded an initial average model. We simulated near vertical-critical refractions from a set of smaller arrays. The parameters were twelve v-points and their corresponding six z-points. The "real" model and the results of three iterations are shown in figure 5.

REAL DATA:

In this very simple example we chose the travel times from an explosion and the reported results shown in figure 6. Two steps were required. In the first the common depth for the two media was inverted to obtain an average depth. In the second, we added points to the discontinuity in order that it could basculate. The result of this example is given in figure 7.

CONCLUSIONS:

We have shown by means of very simple formula that the problem of finding boundaries between media can be solved with a high degree of accuracy, while keeping the number of parameters low. Both velocity and depth parameters may be used if lateral heterogeneity is present. Three-D extension of the method is highly desirable.

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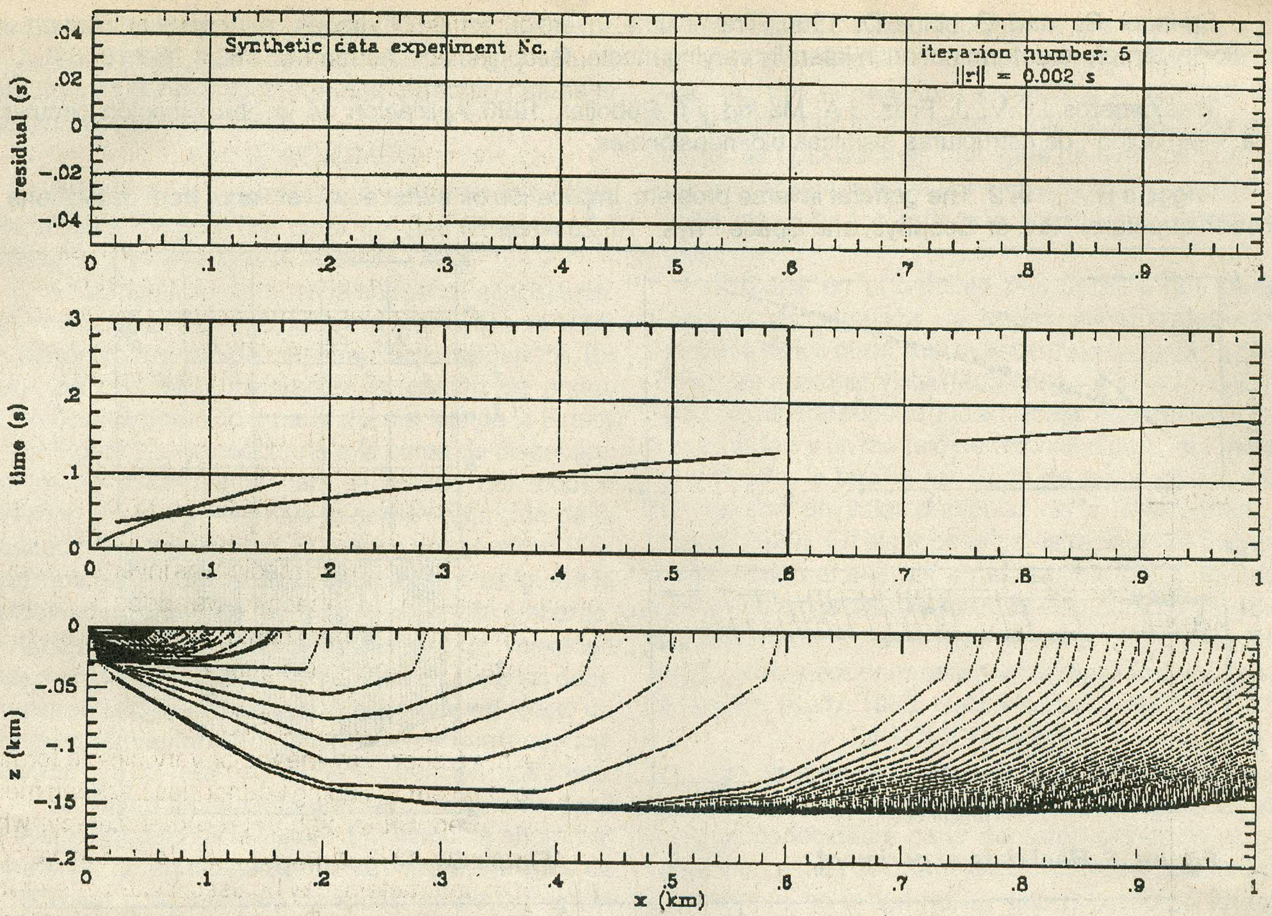

Figure 4. Syntetic data experiment. The true model is a basin.

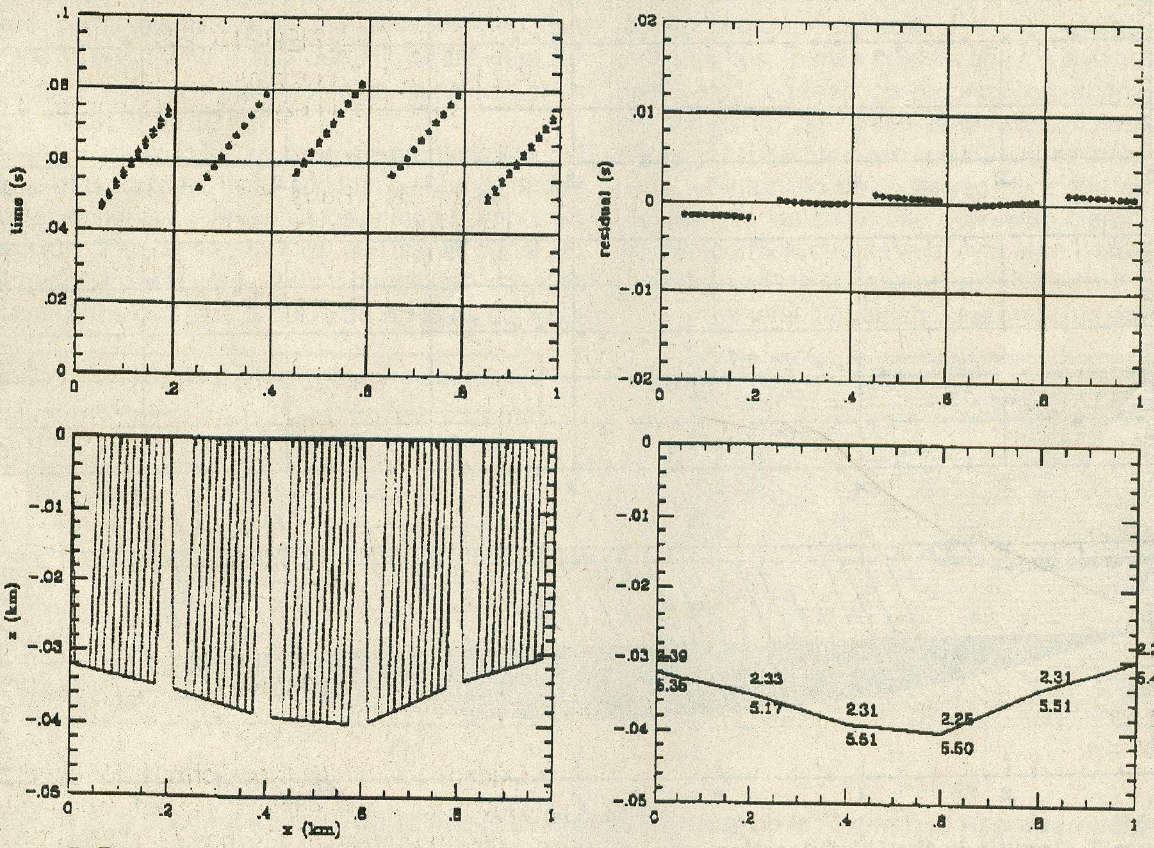

Figure 5. Results in the third iteration.

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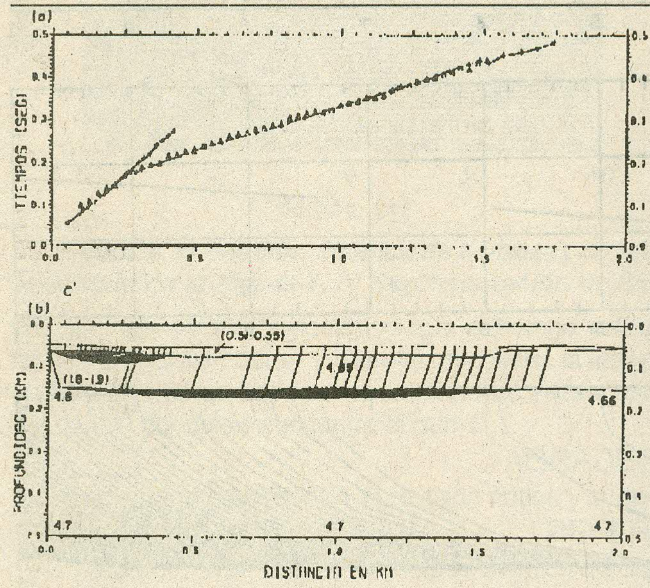

Figure 6. Real data experiment.

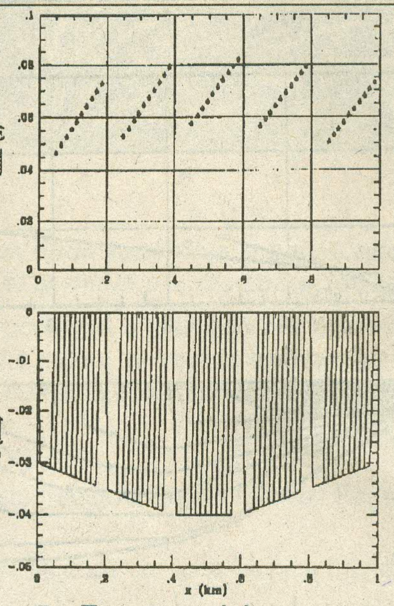

Figure 5a. Target model.

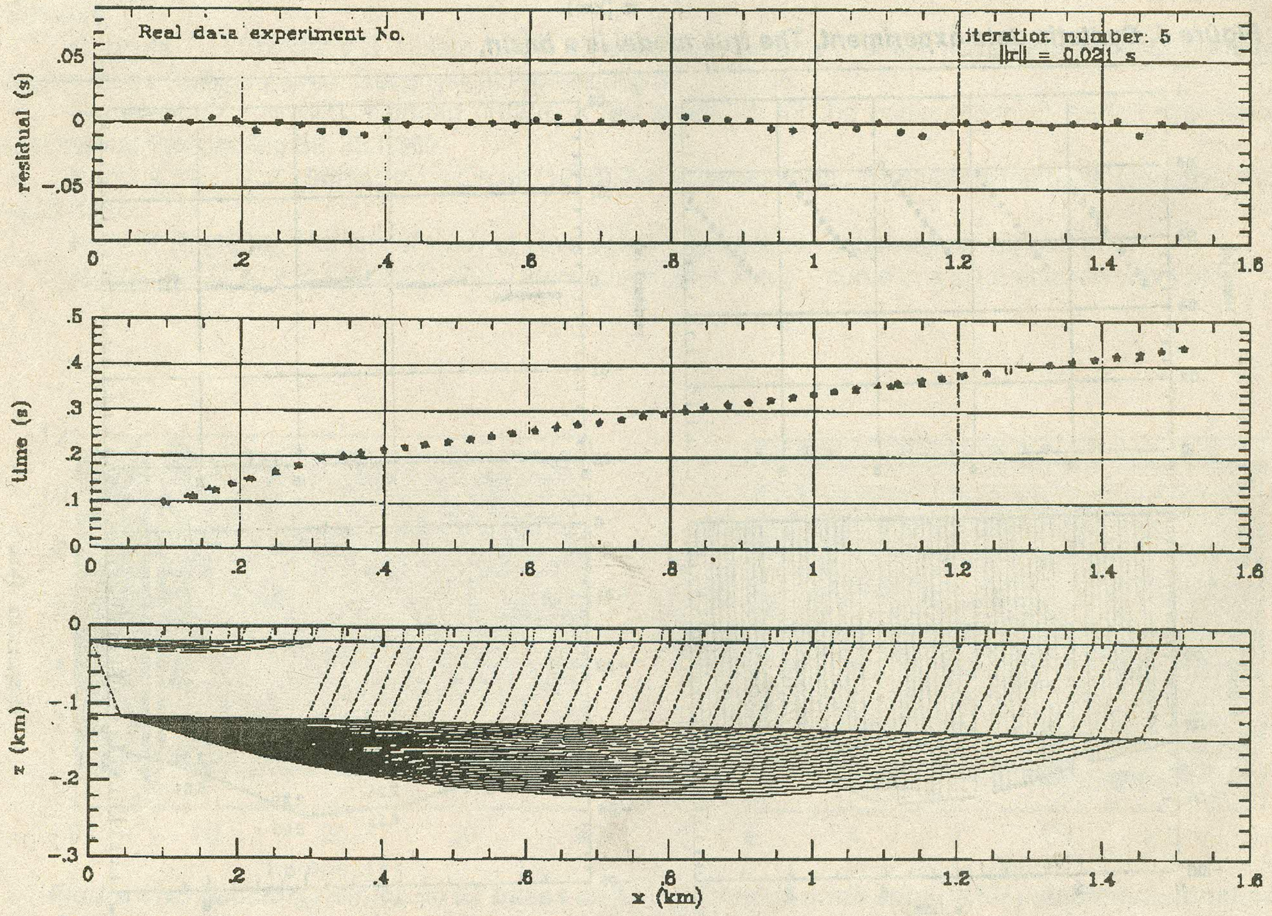

Figure 7. Results in the third iteration, second step.